

nag_bessel_k1 (s18adc)

1. Purpose

nag_bessel_k1 (s18adc) returns the value of the modified Bessel function, $K_1(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>
```

```
double nag_bessel_k1(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the modified Bessel function of the second kind, $K_1(x)$.

The function is based on Chebyshev expansions.

For x near zero, $K_1(x) \simeq 1/x$. For very small x on some machines, it is impossible to calculate $1/x$ without overflow and the function must fail.

4. Parameters

x

Input: the argument x of the function.

Constraint: $x > 0.0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: $x = \langle \text{value} \rangle$.

K_0 is undefined and the function returns zero.

NE_REAL_ARG_TOO_SMALL

On entry, **x** must be greater than $\langle \text{value} \rangle$: $x = \langle \text{value} \rangle$.

x is too small, there is a danger of overflow and the function returns approximately the largest representable value.

6. Further Comments

6.1. Accuracy

Let δ and ϵ be the relative errors in the argument and result respectively.

If δ is somewhat larger than the **machine precision** (i.e., if δ is due to data errors etc.), then ϵ and δ are approximately related by $\epsilon \simeq |(xK_0(x) - K_1(x))/K_1(x)| \delta$.

However, if δ is of the same order as the **machine precision**, then rounding errors could make ϵ slightly larger than the above relation predicts.

For small x , $\epsilon \simeq \delta$ and there is no amplification of errors.

For large x , $\epsilon \simeq x\delta$ and we have strong amplification of the relative error. Eventually K_1 , which is asymptotically given by $e^{-x}/\sqrt{x}$, becomes so small that it cannot be calculated without underflow and hence the function will return zero. Note that for large x the errors will be dominated by those of the **math library** function exp.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9 p 374.

7. See Also

nag_bessel_k0 (s18acc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_bessel_k1(s18adc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
    double x, y;

    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vprintf("s18adc Example Program Results\\n");
    Vprintf("      x      y\\n");
    while (scanf("%lf", &x) != EOF)
    {
        y = s18adc(x, NAGERR_DEFAULT);
        Vprintf("%12.3e%12.3e\\n", x, y);
    }
    exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s18adc Example Program Data
      0.4
      0.6
      1.4
      1.6
      2.5
      3.5
      6.0
      8.0
     10.0
    1000.0
```

8.3. Program Results

```
s18adc Example Program Results
      x      y
  4.000e-01  2.184e+00
  6.000e-01  1.303e+00
  1.400e+00  3.208e-01
  1.600e+00  2.406e-01
  2.500e+00  7.389e-02
  3.500e+00  2.224e-02
  6.000e+00  1.344e-03
  8.000e+00  1.554e-04
  1.000e+01  1.865e-05
  1.000e+03  0.000e+00
```
